Name ____________________ Period __________ Date __________

8-5
STUDENT PACKET

MATHLINKS GRADE 8
STUDENT PACKET 5
EXPRESSIONS AND EQUATIONS 2

WORD BANK

Word or Phrase	Definition or Explanation	Example or Picture
additive inverse property		
distributive property		
equation		
evaluate		
expression		
simplify		
solve (an equation)		
substitution		
terms		
variable		

CUPS AND COUNTERS EXPRESSIONS

Summary (Ready)	**Goals (Set)**
We will use a model to write and simplify expressions.	• Use variables in expressions. • Use the distributive property. • Use the additive inverse property. • Practice combining like terms.

Warmup (Go)

1. In your own words, explain the difference between an expression and an equation.

Translate each verbal expression into a variable expression.

2. Cindy has m books on her shelf and Mindy has n books on her shelf. How many books do they have together?

3. Alexi has p video games and Jin also has p video games. How many video games do they have together?

Evaluate each expression for $a = 3$, $b = -2$, $c = -4$.

4. $a + bc$

5. $\frac{ac}{b}$

6. $a(c - b)$

7. $b^2 + c$

EXPRESSIONS WITH CUPS AND COUNTERS

Build and draw each expression. Write each expression in its simplest form. Evaluate each expression for the given values of *x*. Show all work.

A “cup” represents an unknown value, such as *x*, and is represented by this picture: V	An “upside-down cup” represents the <u>opposite</u> of an unknown value, such as -*x*, and is represented by this picture: Λ

Expression	Picture	Evaluate for given values of *x*: Input (*x*)	Output
1. $x + (-3)$ or $x - 3$	V – – –	4	$4 - 3 = 1$
		-4	
2. $-x + 3$	Λ + + +	4	
		-4	
3. $3x + (-4)$ or $3x -$ _____		2	
		-2	

EXPRESSIONS WITH CUPS AND COUNTERS (Continued)

Build and draw each expression. Write each expression in its simplest form. Evaluate each expression for the given values of x. Show all work.

Expression	Picture	Evaluate for given values of x

4. $2(x + 1)$

or

____ + _____

Think: "2 groups of (x + 1)"

Input (x)	Output
0	
3	
-3	

5. $- 2(x + 1)$

or

____

First think of 2(x + 1)…

V V

\+ +

then draw its opposite, - 2(x + 1).

Input (x)	Output
0	
5	
-5	

6. $x + (-x)$

or

____

Input (x)	Output
0	
10	
-10	

EXPRESSIONS WITH CUPS AND COUNTERS (Continued)

Build and draw each expression. Write each expression in its simplest form. Evaluate each expression for the given values of x. Show all work.

Expression	Picture	Evaluate for given values of x

Expression	Picture	Input (x)	Output
7. $4x + 3 + x - 1$ or		0	
		2	
		-2	

Expression	Picture	Input (x)	Output
8. $3(x + 2) - (x + 2)$ or		0	
		3	
		-3	

EXPRESSIONS PRACTICE

Draw a picture for each expression. Write each expression in its simplest form. Evaluate each expression for the given values of x. Show all work.

Expression	Picture	Evaluate for given values of x	
		Input (x)	Output
1. $3(x - 1)$		4	
		-2	
		Input (x)	Output
2. $-(2x + 1)$		7	
		-5	
		Input (x)	Output
3. $2x + 4 - 2x + 2$		0	
		-3	
		Input (x)	Output
4. $(2x + 4) - (2x + 2)$		0	
		-3	

EXPRESSIONS PRACTICE (Continued)

Draw a picture for each expression. Write each expression in its simplest form. Evaluate each expression for the given values of x. Show all work.

Expression	Picture	Input (x)	Output
5. $x + 1 - x + 1$		13	
		-15	
6. $(x + 1) - (x + 1)$		7	
		-18	
7. $-(2x + 3)$		2	
		-1	
8. $-2(x + 3)$		0	
		-3	

TRANSPORTING PUPPIES

A mother dog has 8 puppies. She goes back and forth to bring them from a dog house to a water bowl.

- First predict the number of one-way trips for each problem below.
- Then draw simple diagrams using arrows and other symbols to illustrate the situations, and answer the questions.

1. If she retrieves them **one at a time**, how many one-way trips must she make to get all 8 puppies plus herself from the dog house to the water bowl? Prediction: __________ Answer:	2. If she retrieves them **two at a time**, how many one-way trips must she make to get all 8 puppies plus herself from the dog house to the water bowl? Prediction: __________ Answer:

INTRODUCTION TO THE LAKE PROBLEM

Use the information to answer the questions and to determine how many one-way trips are needed to get everyone across the lake.

- Six adults and two children need to cross a lake to get from the hiking trail back to their tents.
- They have a canoe that can hold **one child alone**, OR **two children together**, OR **one adult alone**.
- Everyone is able to paddle the canoe. The only way to get across the lake is to use the canoe.

1. Who must get across the lake?

2. How many children can fit in the canoe at any one time?

3. How many adults can fit in the boat at any one time?

4. Does the trip from the hiking trail to the tents count as one trip?

5. Does the trip from the tents back to the hiking trail count as one trip?

6. What is the question we are trying to answer?

7. Draw very simple pictures/sketches that you could use to represent the three possibilities for people in a canoe at one time.

One Child	Two Children	One Adult

SOLVING THE LAKE PROBLEM

1. Record your diagram here for solving the lake problem with any symbols or notations that are helpful. Use additional paper if needed.

2. Look closely at your diagram. Do you see any patterns? Does anything seem to be happening regularly, over and over again? Circle a repeating pattern if you see one. Write your observations below.

Use your diagram as needed to determine the number of one-way trips necessary to take each combination of people across the lake.

3. 4 adults and 2 children	4. 2 adults and 2 children
5. 0 adults and 2 children	6. 20 adults and 2 children
7. 100 adults and 2 children	8. *a* adults and 2 children
9. If there were some adults and 2 children and it took 201 one-way trips to get everyone across the lake, how many adults must have been in the group?	

NUMBER TRICKS

Summary (Ready)	Goals (Set)
We will perform mathematical number tricks and use algebraic expressions to show how they work.	• Use algebraic expressions to generalize patterns. • Apply properties of arithmetic to simplify algebraic expressions. • Evaluate expressions with rational numbers. • Translate verbal expressions as algebraic expressions.

Warmup (Go)

1. What is a natural number? ____

2. Perform the number trick below. We will call this Number Trick 1.

Step	Words	Numbers	
1	Choose a single digit number.		
2	Multiply your number by 2.		
3	Add 8 to your answer.		
4	Divide your answer by 2.		
5	Subtract your original number (in step 1) from your answer.		

3. What is your final result? _____ Compare answers with your classmates' answers. Do you think this trick will work for all numbers? Explain.

NUMBER TRICK 2

1. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a single digit natural number.		**V**	n
2	Add the number to itself.		**V V**	$n + n$ = _____
3	Add 3.		**V V + + +**	$2n$ + _____
4	Double the result.			$2(2n + 3)$ = _______
5	Subtract your original number.			
6	Divide by 3.			
7	Subtract 2.			
8	What number do you have now?			

2. What is the number trick? ____

____

3. Does this trick always work? Explain. ____

____

NUMBER TRICK 3

1. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a natural number.			
2	Multiply by 4.			
3	Add 6.			
4	Multiply by $\frac{1}{2}$.			
5	Add 5.			
6	Divide by 2.			
7	Subtract 4.			
8	Add your original number.			
9	What number do you have now?			

2. What is the number trick? ____

____

3. Does this trick always work? Explain. ____

____

NUMBER TRICKS 4 AND 5

1. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a number.		**V**	n
2	Add 4.		**V** + + + +	$n+4$
3	Multiply by 2.			
4	Subtract 8.			
5	Divide by 2.			
6	What is the result?			

2. What is the number trick? ____

3. Does this always work? Explain. ______________________________________

4. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a number.		**V**	n
2	Add 3.			
3	Multiply by 2.			
4	Subtract 4.			
5	Multiply by $\frac{1}{2}$.			
6	Subtract the original number.			
7	What is the result?			

5. What is the number trick? ____

6. Does this trick always work? Explain. ____________________________________

CUPS AND COUNTERS EQUATIONS 1

Summary (Ready)	Goals (Set)
We will solve linear equations with one variable using a visual model and record the steps with pictures and algebraic symbols.	• Use a model to solve multistep algebraic equations. • Use algebraic notation to write and justify steps. • Use substitution to check solutions. • Recognize that linear equations may have no solutions, one solution, or infinitely many solutions.

Warmup (Go)

Use a mental strategy or cover-up strategy to solve for x, check your solution, and explain how you solved the equation.

1. $8 = 20 - 2x$	2. $-3(x + 6) = -30$
Solution and check:	Solution and check:
Explanation.	Explanation.

INTRODUCTION TO EQUATIONS WITH CUPS AND COUNTERS

Follow your teacher's instructions to build and record equations. Use a mental strategy to check the solution.

1.

Picture	Equation
V = + + +	____ = ____

2.

Picture	Equation
=	$6 = x + 2$ $4 = x$ Check: $6 =$ ____ $+ 2$

3.

Picture	Equation
=	$2x = 6$ $x =$ ____ Check: 2(____) $= 6$

4.

Picture	Equation
=	$2x - 3 = 11$ $2x =$ _____ $x =$ _____ Check: 2(____) – 3 $= 11$

SOLVING EQUATIONS USING A MODEL

Solve each equation by building, drawing, and recording each step. Then check your solution.

1.

Picture	Equation/Steps	What did you do?
=	$5x + 4 = 2x - 2$	
	=	
	=	
	=	

Check your solution using substitution:

2.

Picture	Equation/Steps	What did you do?
=	$2(x - 1) = 2x + 1 + x$	
	=	
	=	
	=	

Check your solution using substitution:

SOLVING EQUATIONS USING A MODEL (Continued)

Solve each equation by building, drawing, and recording each step. Then check your solution.

3.

Picture	Equation/Steps	What did you do?
=	$2(x+3) = 4(x-1)$	
	=	
	=	
	=	

Check your solution using substitution:

4.

Picture	Equation/Steps	What did you do?
=	$-1 + x + 2 = 4x + 1$	
	=	
	=	
	=	

Check your solution using substitution:

SOLVING EQUATIONS USING A MODEL (Continued)

Solve each equation by building, drawing, and recording each step. Then check your solution.

5.

Picture	Equation/Steps	What did you do?
=	$2(x + 4) = 3x + 2$	
	=	
	=	
	=	

Check your solution using substitution:

6.

Picture	Equation/Steps	What did you do?
=	$-4 + x + 1 = 5x - 7$	
	=	
	=	
	=	

Check your solution using substitution:

SOLVING EQUATIONS USING A MODEL (Continued)

Solve each equation by building, drawing, and recording each step. Then check your solution.

7.

Picture	Equation/Steps	What did you do?
=	$6(x-2) = 4x - 6$	
	=	
	=	
	=	

Check your solution using substitution:

8.

Picture	Equation/Steps	What did you do?
=	$1 + x = 3(x-2) - 2$	
	=	
	=	
	=	

Check your solution using substitution:

SOLVING EQUATIONS CHALLENGE

Solve each equation by building, drawing, and recording each step. Then check your solution.

1.

Picture	Equation/Steps	What did you do?
=	$2(x - 1) + 2x - 1 \;=\; 4x - 3$	
	=	
	=	
	=	

Why is this an example of a linear equation with infinitely many solutions?

2.

Picture	Equation/Steps	What did you do?
=	$2(2x + 1) \;=\; 4x + 5$	
	=	
	=	
	=	

Why is this an example of a linear equation with no solutions?

SOLVING PROBLEMS USING MULTIPLE METHODS

For each problem, first solve it using any method. Then translate the problem into an equation, and solve the equation using any strategy (use a cups and counters picture if needed).

1. The perimeter of a rectangle is 14 cm. Its length is 3 cm. What is its width?	2. The perimeter of an isosceles triangle is 15 mm. The two congruent sides are each 3 mm longer than the shorter side. How long is the shorter side?
Method 1 (your choice)	Method 1 (your choice)
Solution:	Solution:
Method 2 (using algebra)	Method 2 (using algebra)
let P = perimeter, ℓ = length, w = width	let x = the length of the shorter side
Equation: $P = 2\ell + 2w$	Equation:
Solution:	Solution:

SKILL BUILDERS, VOCABULARY, AND REVIEW

SKILL BUILDER 1

Write the property of arithmetic illustrated by each equation.

1. (4)(16) = 4(10 + 6) ______________________________

2. (12 • 3) • 5 = 12 • (3 • 5) ______________________________

3. 8(5 + 6) = 8(6 + 5)______________________________

Write an equation that illustrates each property indicated.

	Property of Arithmetic	Equation
4.	distributive property	
5.	multiplicative identity property	
6.	additive inverse property	
7.	additive identity property	
8.	commutative property of addition	
9.	associative property of multiplication	

Compute.

10. $-6-(-8)$	11. $8-12$	12. $5-(-6)$
13. $-13-15$	14. $-18-9$	15. $17-4$
16. $7-(-7)+(-7)$	17. $100-(-25)$	18. $35-(-15)$
19. $(-1)(-2)(-3)(-4)$	20. $-36 \div (-6) - 6$	21. $\dfrac{-36}{-6-6}$

SKILL BUILDER 2

Translate each verbal expression into a variable expression

1. Three times the sum of a number *v* and 9.	2. The sum of 9 and three times a number *v*.

3. Translate the verbal inequality into symbols, solve it mentally, and graph the solution(s).

a. Words: A number times 3 is at least 18.	b. Symbols (choose a variable):
c. Solution(s):	d. Graph: 

Use mental math to solve the following inequalities. Then graph the solution(s). Check a number in the solution set to verify that it makes the inequality true.

4. Inequality: $6x > -30$ Think (write in words): Solution: Graph:  Check a number in the solution set.	5. Inequality: $x + (-4) \leq -2$ Think (write in words): Solution: Graph:  Check a number in the solution set.

Solve mentally.

6. $\frac{1}{4}(x - 5) = 10$	7. $\frac{48}{-4+(-x)} = 6$

SKILL BUILDER 3

Compute.

1. $\frac{5}{6}+\frac{4}{8}$	2. $2-1\frac{2}{3}$	3. $\frac{1}{3}\bullet 3$
4. $\frac{15}{16}\div\frac{5}{8}$	5. $\frac{5}{6}\bullet\frac{24}{25}$	6. $\frac{4}{6}\bullet\frac{6}{4}$

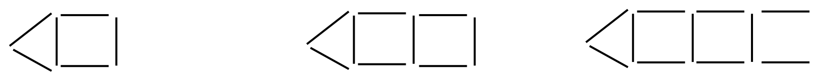
Simplify.

7. $4\bullet\frac{1}{4}n$	8. $\frac{1}{6}\bullet 6m$	9. $\frac{3}{2}\bullet\frac{2}{3}v$	10. $-\frac{3}{5}\bullet\left(-\frac{5}{3}w\right)$

11. What is the same about the results in the problems above? Explain why this occurs.

SKILL BUILDER 4

Draw the next step suggested by this pattern. Then complete the table and find a rule for the number of toothpicks at step *n*.



step1 step 2 step 3 step 4

Step #	0	1	2	3	4	5	50	*n*
Number of toothpicks								
Expressions								

1. Label the horizontal and vertical axes and graph the data points.

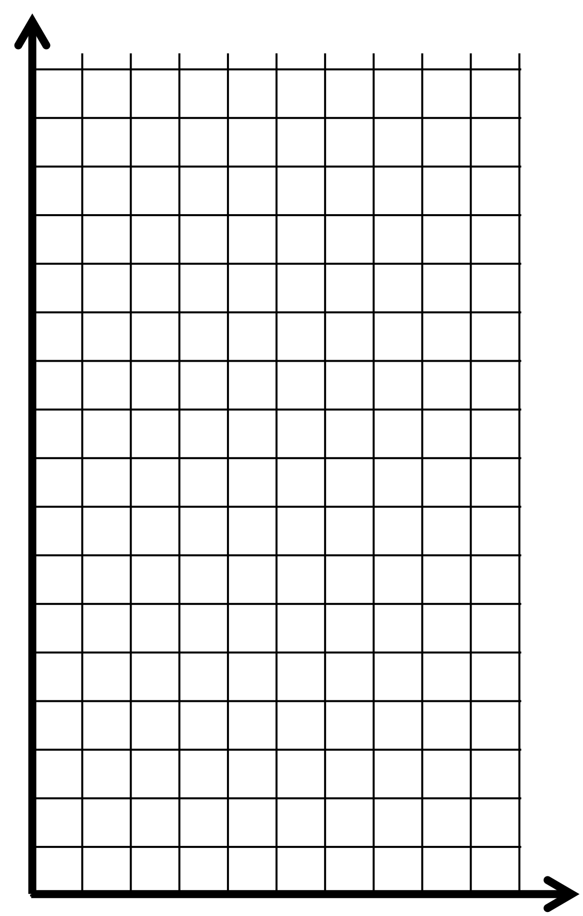
2. Recursive Rule:

   Start with ______ toothpicks,

   and then ____________ each step.

3. Explicit Rule: Explain what to do to the input number at each step to get the corresponding output number.

4. In which step number are there exactly 84 toothpicks?



SKILL BUILDER 5

For each equation, find the output values (*y*) for the given input values (*x*). Graph the ordered pairs for each equation using a different color pencil. Connect the points for each equation.

1. $y = \frac{1}{2}x$

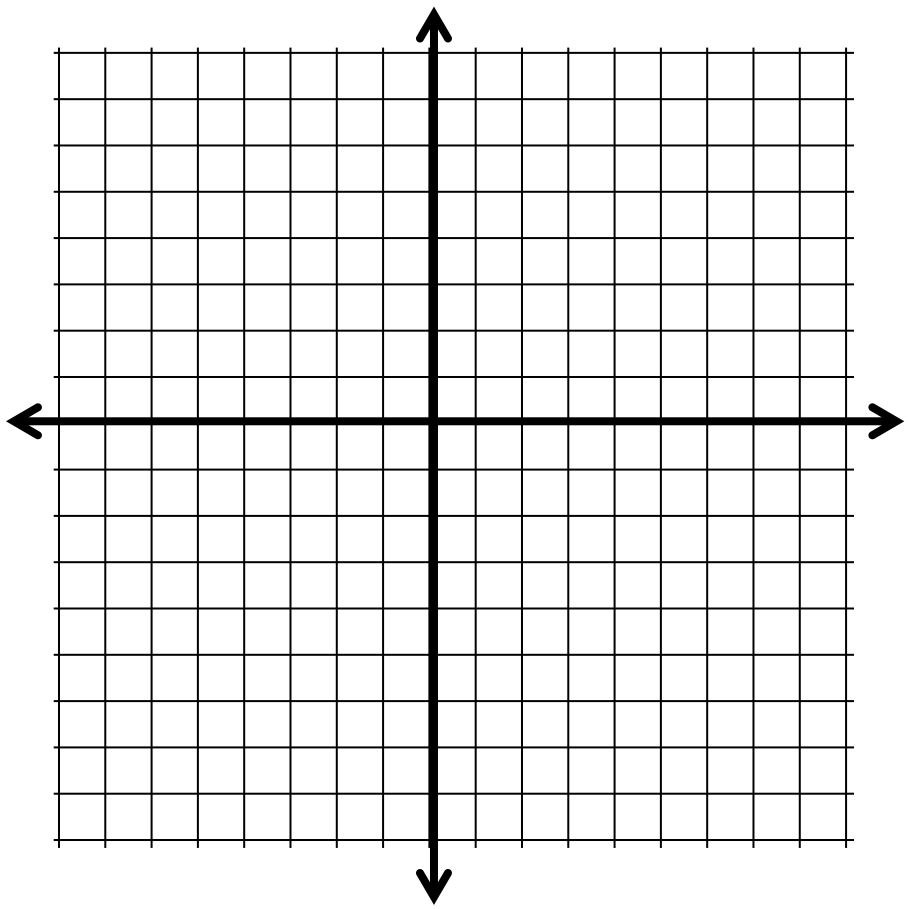
x	y
0	
8	
-4	

2. $y = \frac{1}{2}x + 3$

x	y
0	
2	
-4	

3. $y = \frac{1}{2}x - 4$

x	y
0	
6	
-2	



4. How are the graphs of the equations the same?

5. How are the graphs of the equations different?

SKILL BUILDER 6

Find the missing values in each input-output table, and write an explicit rule for the data.

1.

x	y
1	5
2	9
3	13
	17
5	
6	
Rule: y = ________	

2.

x	y
-2	
0	0
3	9
7	21
	36
	-3
Rule: y = ________	

3.

x	y
-14	7
4	-2
	-8
22	-11
	14
	-1
Rule: y = ________	

Build and draw each expression, then simplify and evaluate the expression for the given values.

Expression	Picture	Evaluate for given x values	
4. $-2x + 4 + x + 2$ or		Input (x)	Output
		2	
		-6	
5. $-(x + 4) + (x + 2)$ or		Input (x)	Output
		2	
		-6	

SKILL BUILDER 7

1. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a number.		**V**	n
2	Multiply by 5.			
3	Add 2.			
4	Double it.			
5	Subtract 4.			
6	What is the result?			

2. Describe the number trick and explain why it always works.

3. Perform the number trick below.

Step	Words	Numbers	Pictures	Symbols
1	Choose a whole number.		**V**	n
2	Write the number that is one more than your original number.			
3	Write the number that is two more than your original number.			
4	Write the sum of these 3 consecutive whole numbers.			
5	Divide the sum by 3.			
6	What is the result?			

4. Describe the number trick and explain why it always works.

SKILL BUILDER 8

Build, draw, record, and solve each equation. Then check your solution.

1.

Picture	Equation/Steps	What did you do?
	$3(x-1) = 3x + 1 + x - 5$	
	=	
	=	
	=	
Check your solution using substitution:		

2.

Picture	Equation/Steps	What did you do?
	$3(x-1) = 3x - 1 + x$	
	=	
	=	
	=	
Check your solution using substitution:		

SKILL BUILDER 9

Build, draw, record, and solve each equation. Then check your solution.

1.

Picture	Equation/Steps	What did you do?
	$4x - 8 + 2x = 7x - x - 8$	
	=	
	=	
	=	

Check your solution using substitution:

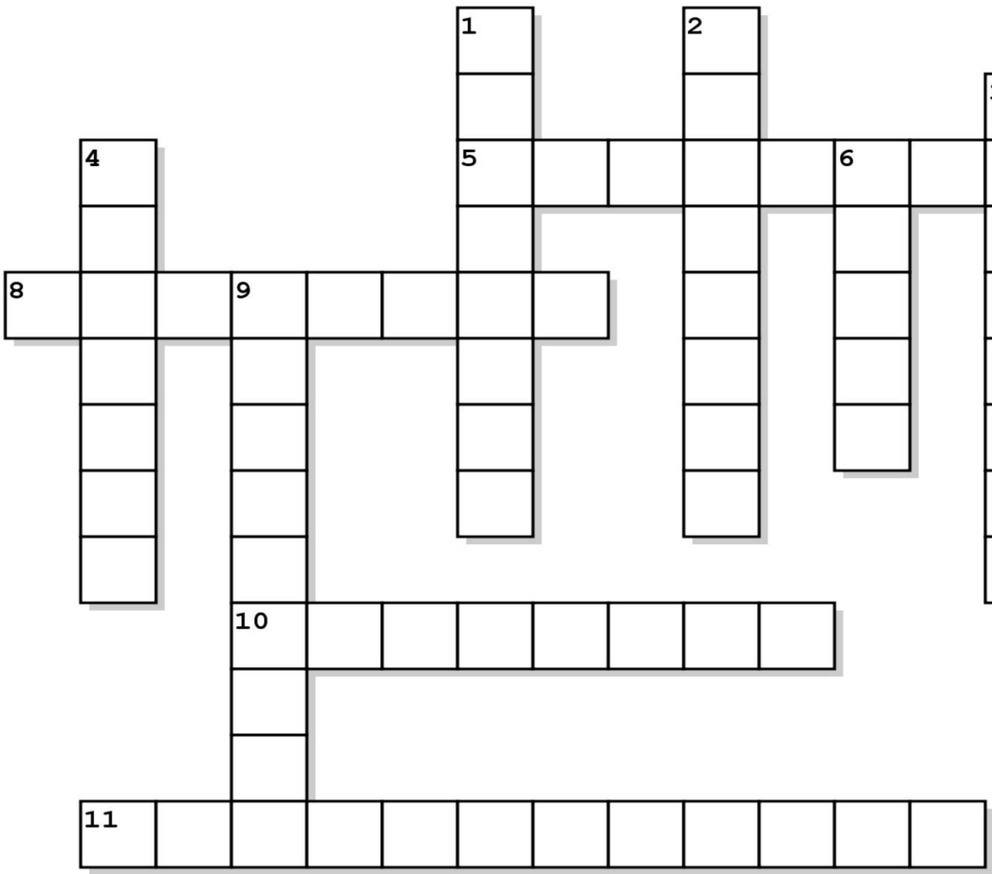
2.

Picture	Equation/Steps	What did you do?
	$3(2x + 1) - 6 = 2(3x - 1)$	
	=	
	=	
	=	

Check your solution using substitution:

FOCUS ON VOCABULARY

Use vocabulary from this packet to complete the crossword puzzle.



Across	Down
5 $3x - 6$, $2x$, or x^2, for example	1 $-5 + 0 = -5$ illustrates the additive ____ property
8 substitute a number to find the value of an expression	2 quantity whose value is not specified
10 $8 - 3 = 3x - 4$, for example	3 convert an expression to a simpler form
11 property illustrated by $3(6 + x) = 18 + 3x$	4 $-5 + 5 = 0$ illustrates the additive ____ property
	6 find the value for a variable that makes an equation true
	7 the numbers 1, 2, 3, …
	9 $5x$ and $-3x$, for example (two words)

SELECTED RESPONSE

Show your work on a separate sheet of paper and choose the best answer(s).

1. Which expression does <u>not</u> match the following picture?

V ++ V ++

A. $2x + 4$ B. $2(x + 2)$ C. $2x + 2$ D. $x + 2 + x + 2$

2. Which expression does <u>not</u> match the following picture?

Λ Λ Λ
– – –

A. $3x – 3$ B. $-3x + (-3)$ C. $-3x – 3$ D. $-3(x + 3)$

3. Which of the following is a solution to this equation: $2(x – 6) = 5x + 9$

A. -1 B. 1 C. -7 D. 7

4. Which would be the least productive first step in solving the following equation *<u>when using cups and counters</u>*: $8x + 6 = 12x – 7$

A. Add 8 negative cups to both sides B. Add 6 negative counters to both sides

C. Add 12 negative cups to both sides D. Add 7 negative counters to both sides.

5. What is the last step in the number trick so that the end result is the same as the original number?

Steps	Directions
1	Choose a single-digit natural number.
2	Add 5.
3	Multiply by 2.
4	Subtract your original number.
5	

A. Subtract 5 B. Add 5 C. Subtract 10 D. Add 10

KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

5.1 Cups and Counters Expressions

Draw each expression with a cups and counters diagram, and then evaluate each for $x = -2$

1. $3x + 2$
2. $3(x + 2)$
3. $-(3x + 2)$

5.2 Number Tricks

4. Perform the number trick below.

Steps	Words	Numbers	Pictures	Symbols
1	Choose a single digit number.			
2	Multiply by 2.			
3	Multiply by 6.			
4	Add 6.			
5	Divide by 2			
6	Divide by 3.			
7	Subtract 1			
8	What number do you have now?			
What is the number trick?				

5.3 Cups and Counters Equations 1

Draw each equation with a cups and counters diagram and then solve. Be sure to show all steps. Check your solutions.

5. $5x + 5 = 3x + 11$
6. $2(x – 3) = 4(x + 2)$

HOME-SCHOOL CONNECTION

Here are some questions to review with your young mathematician.

1. Draw the expression $4(x - 1)$ and then evaluate it for $x = -5$

2. Perform the number trick below.

Steps	Words	Numbers	Pictures	Symbols
1	Choose an even number			
2	Divide by 2.			
3	Multiply by 4.			
4	What number do you have now?			
What is the number trick?				

3. Solve the equation $3(x - 1) = 5x + 7$ using a cups and counters sketch.

Parent (or Guardian) Signature ____________________________

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COMMON CORE STATE STANDARDS – MATHEMATICS

STANDARDS FOR MATHEMATICAL CONTENT	
6.EE.3*	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*
6.EE.4*	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
6.EE.5*	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? ~~Use substitution to determine whether a given number in a specified set makes an equation or inequality true.~~
6.EE.6*	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.7*	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
7.EE.4a*	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
8.EE.7.a	Solve linear equations in one variable. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
8.EE.7b	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

*Review of content essential for success in 8th grade.

STANDARDS FOR MATHEMATICAL PRACTICE	
MP1	Make sense of problems and persevere in solving them
MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.
MP5	Use appropriate tools strategically.
MP7	Look for and make use of structure.

9 7 8 1 6 1 4 4 5 2 1 3 3