2/3

Problem of the Month

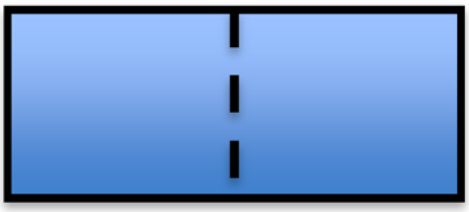
3/4

Fractured Numbers

Level A

Rosita has made a puzzle. She takes a whole rectangle like the one below.

She cuts the whole into half.



She takes that half and cuts it in half.

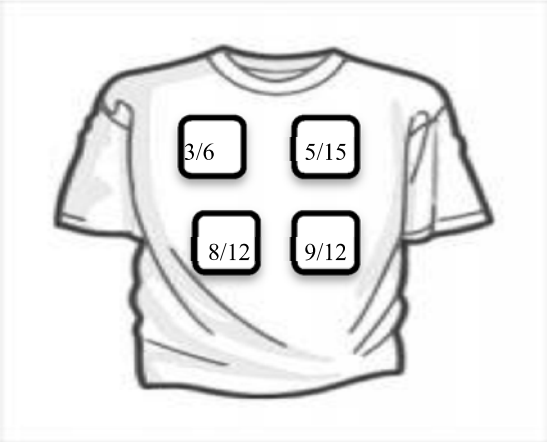
Finally she takes the small piece she cut and cuts that in half.

Now she has the piece she wants. How many of these small pieces can she put together to make the same size rectangle that she started with? Explain how you know.

If she had cut her small piece in half again, how many of those pieces would be needed to make the first rectangle?

What if she kept going? How would the pieces get smaller?

Level B



Jeff is playing a new video game. The goal is to explore a cave with many levels and collect gold coins. In order to get through a door to the next level, you must master a secret code. Jeff's video character has four pockets on his jacket. The top left pocket is marked 3/6, the top right pocket is marked 5/15, the lower left pocket is marked 8/12 and the lower right pocket is marked 9/12.

Above the cave door at each level is a number card. To open the door, he must take the number card from the door and put it in the correct pocket. If he puts it in the wrong pocket, he loses one of his 3 lives. Here are the cave doors he must travel through.

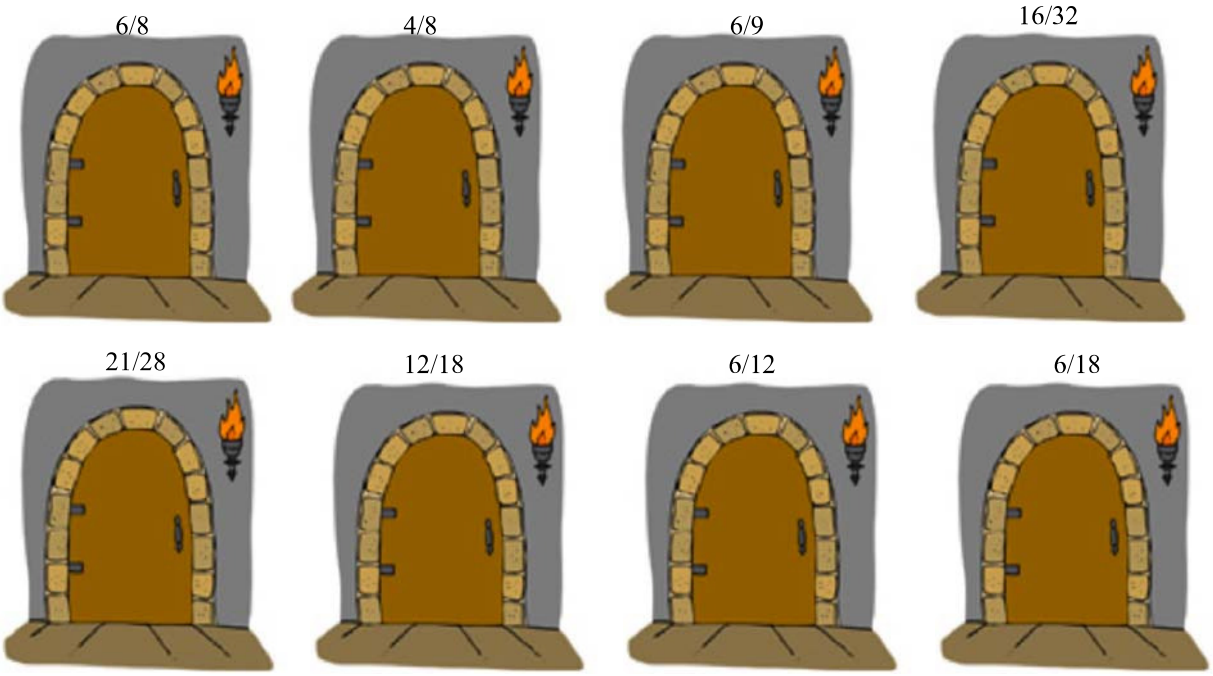


Figure out the secret code to get through all the doors. Write Jeff a note letting him know which numbers go into which pocket. Explain to Jeff how you know for sure. Remember, if you get it wrong he loses a life!

Level C

Roxie is a show dog. Her trainer wants her to have a beautiful and brilliant coat. The veterinarian suggested a special diet for the trainer to follow. Each feeding, Roxie eats 2/3 of a can of wet dog food, 1/8 of a bag of dry dog food, and 3/5 a patty of special meat. The special meat comes in a package of 6 patties. Roxie has two meals a day.

The dog is completely out of food. The trainer goes to the store and buys 24 cans of wet food, 4 bags of dry food and 3 packages of meat.

How many days will the dog be fed before the trainer needs to buy any more food?

Which type of dog food will the trainer run out of first? Explain.

How much of the other two types of dog food will be left after the first type of dog food runs out?

The trainer wants to plan better. She goes to the store on the day she ran out of the first type of dog food. She decides to buy enough dog food to last 90 days. Knowing what she already has in the house, how much more of each type of dog food does she need to buy in order to use up all the food in 90 days? Is it possible? Explain.

What is the minimum amount of food the trainer could buy such that the dog would finish all of it after a certain number of meals? Explain.

Level D

Tracy gave the following explanation to her friend.

I have found a special set of fractions that I call Consecutive Sum Fractions. To be a Consecutive Sum Fraction, it must be equivalent to 1. You start with the unit fraction and then add each larger fraction, the next natural number numerator of the same denominator, until you reach exactly one. For example, 15/15 is a Consecutive Sum Fraction, because;

$1/15 + 2/15 + 3/15 + 4/15 + 5/15 = 1$

Not all equivalent fractions equal to one are Consecutive Sum Fractions. For example consider 12/12. The follow partial sum approaches 1.

$1/12 + 2/12 + 3/12 + 4/12 = 10/12$

But it is still smaller than 1. If we add the next consecutive fraction, we get a fraction larger than 1.

$1/12 + 2/12 + 3/12 + 4/12 + 5/12 = 15/12$

Therefore, 12/12 is not a Consecutive Sum Fraction.

Which fractions are Consecutive Sum Fractions?

How can you determine or predict which will be Consecutive Sum Fractions?

Determine a means to generate all fractions that are Consecutive Sum Fractions.

Level E

Cheryl's mother baked a large rectangular pan of brownies for Cheryl and her two friends, Joanne and Richard, to share after school. Cheryl and Joanne got to the house at the same time. They knew Richard was coming over later.

They cut the brownie into three equal size pieces. Cheryl took one of the pieces and Joanne took another, leaving the third piece for Richard. Cheryl and Joanne each began to eat their share of brownies. When they were finished, Richard had still not shown up.

They were both still hungry so they decided to take Richard's piece and divide it into thirds again. Cheryl and Joanne began to eat their new "one-third" slices, leaving Richard with only one-third of his original piece.

Still Richard did not show, so the two friends decided to cut his remaining piece in thirds again. They set aside one of the cut pieces for Richard, and ate the other two.

If Richard never comes over to the house and the two friends continue their process of eating and dividing the remaining slices, how much will they each eat?

- Represent your answer in an equation.
- What can you conclude from an infinite sum?
- Suppose there were four friends and only three showed up to eat the brownies originally cut into fourths. If a similar process occurred how much would each of the three friends eat? Represent your conclusion in an equation with an infinite sum.
- Generalize your finding about similar sets of infinite sums.