

Lesson 3: Numbers in Exponential Form Raised to a Power

Classwork

For any number x and any positive integers m and n ,

$$(x^m)^n = x^{mn}$$

because

$$\begin{aligned} (x^m)^n &= \underbrace{(x \cdot x \cdots x)}_{m \text{ times}}^n \\ &= \underbrace{(x \cdot x \cdots x)}_{m \text{ times}} \times \cdots \times \underbrace{(x \cdot x \cdots x)}_{m \text{ times}} \quad (n \text{ times}) \\ &= x^{mn} \end{aligned}$$

Exercise 1

$$(15^3)^9 =$$

Exercise 3

$$(3,4^{17})^4 =$$

Exercise 2

$$((-2)^5)^8 =$$

Exercise 4

Let s be a number.

$$(s^{17})^4 =$$

Exercise 5

Sarah wrote that $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y , and positive integer n ,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned} (xy)^n &= \underbrace{(xy) \cdots (xy)}_{n \text{ times}} \\ &= \underbrace{(x \cdot x \cdots x)}_{n \text{ times}} \cdot \underbrace{(y \cdot y \cdots y)}_{n \text{ times}} \\ &= x^n y^n \end{aligned}$$

Exercise 7

$$(11 \times 4)^9 =$$

Exercise 8

$$(3^2 \times 7^4)^5 =$$

Exercise 9

Let a , b , and c be numbers.

$$(3^2 a^4)^5 =$$

Exercise 10

Let x be a number.

$$(5x)^7 =$$

Exercise 11

Let x and y be numbers.

$$(5xy^2)^7 =$$

Exercise 12

Let a , b , and c be numbers.

$$(a^2 b c^3)^4 =$$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n ?

Problem Set

1. Show (prove) in detail why $(2 \cdot 3 \cdot 4)^4 = 2^4 3^4 4^4$.
2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z .
3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers x, y, z , and for any positive integer n .